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# Cardinal Mooney High School Algebra Honors 2 

## Review Packet

## For students entering Algebra 2Honors Summer of 2023

## Directions:

1. In order to take full advantage of the video instruction offered by the web links provided in each section, this packet is best completed while opened on your computer.
2. Complete all exercises and bring this packet to your initial Algebra 2 the first day of school.

The packet will be broken down into 50 points. 30 points for completion and then I will pick 10 random problems and grade for accuracy for 20 points. You may use a calculator (except when noted otherwise); but, show all work used to arrive at your answer. You will be tested on this material the second week of school. Pace yourself, don't wait until the last moment.
3. If you need help on any of the Algebra1 topics in this packet, please refer to the following websites:

- http://coolmath.com/algebra/Algebra1/index.html
- http://www.algebra.com/
- http://www.brightstorm.com/math
- http://www.khanacademy.org


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## 1. Order of Operations

## Order of Operations - Evaluating an Expression

http://www.brightstorm.com/math/algebra/pre-algebra/order-of-operations
http://www.khanacademy.org/video/order-of-operations?playlist=Developmental\ Math
Use PEMDAS (Please Excuse My Dear Aunt Sally):
Step 1. P - Parentheses: start with operations inside grouping symbols (parentheses)
Step 2. E-Exponents: evaluate powers
Step 3. MD - Multiply/Divide: do multiplications and divisions from left to right
Step 4. AS - Add/Subtract: do additions and subtractions from left to right
Ex1)
Ex2)
$15 \cdot 2 \div 6$
$(15 \cdot 2) \div 6$
$30 \div 6=5$
$3 \cdot\left(4^{2}+8\right) \div 4$
$3 \cdot(16+8) \div 4$
$3 \cdot 24 \div 4$ $3 \cdot 6$
18
Ex3)
$\frac{7 \cdot 4}{8+7^{2}-1=} \quad \frac{7 \cdot 4}{8+49-1}$
28
57-1 28 56
1 $\overline{2}$

| Directions: Evaluate each expression using the order of operations. |  |
| :--- | :--- | :--- |
| 1. $17-5 \cdot 4 \div 2$ | 2. $40-32 \div 8+5 \cdot-2$ |
| 3. $35-14 \div 2+8^{2}$ | 4. $(4-7)^{2}-6 \cdot 7+20$ |
| 5. $2\left[18-\left(5+3^{2}\right) \div 7\right]$ | 6. $[(-5+1) \div 2]^{3}$ |

## 2 Solving Equations and Proportions

## Solving Equations

http://www.brightstorm.com/math/algebra/pre-algebra/order-of-operations
http://www.khanacademy.org/video/order-of-operations?playlist=Developmental\ Math

## Ex.1)

$2(3 x-4)=10$
$2(3 x)-2(4)=10$
$6 x-8=10$
$6 x-8+8=10+8$
$6 x=18$
$6 x / 6=18 / 6$
$x=3$

## EX.3)

$$
\begin{aligned}
5 p+10 & =5(p+8) \\
5 p+10 & =5 p+40 \\
5 p-3 p+10 & =5 p-5 p+40 \\
10 & \neq 40 \quad \text { no solution }
\end{aligned}
$$

| Directions: Solve each equation. Show all steps |  |
| :--- | :--- |
| 1. $2(n+2)=4 n+1-2 n$ | 2. $-15-3 w=4 w-2 w$ |
|  |  |
| 3. $2(x+4)=4 x+3-2 x+5$ | 4. $4(2 r-1)=-2(3 r+16)$ |
| 5. $3 a-6 a+2=8 a+20-5 a$ | 6. $-(v+4)+5=4 v+1-5 v$ |

## 2 Solving Equations and Proportions

## Solving Proportions

http://www.brightstorm.com/math/algebra/pre-algebra/order-of-operations
http://www.khanacademy.org/video/order-of-operations?playlist=Developmental\ Math

## Ex1)

$$
\begin{array}{cc}
\frac{5}{2}= & \frac{x+3}{4} \\
2 x+6 & =20 \\
-6 & -6 \\
2 x= & 14 \\
x & 7
\end{array}
$$



## 3) Solving Absolute Value Equations

## Solving Absolute Value Equations

http://www.brightstorm.com/math/algebra/pre-algebra/order-of-operations
http://www.khanacademy.org/video/order-of-operations?playlist=Developmental\ Math

Steps to solving absolute value equations.

1) Isolate the absolute value sign.
2) Take the inside equations and make two equations set equal to their opposites.
3) Solve both equations.
4) Take both answers and check back into the original equations for extraneous solutions.


Solve and check for extraneous solutions.

1. $|x|=7$
2. $|2 x+6|=14$
3. $3|x+2|=18$
4. $|3 x-1|=-6$
5. $|x-5|+2=12$

## 3 Simplifying Radicals

## Use Properties of Radicals to Simplify Expressions

http://www.brightstorm.com/math/algebra/radical-expressions-and-equations/simplifying-radical-expressions
http://www.khanacademy.org/video/simplifying-radicals?playlist=Pre-algebra
Product Property: The square root of a product equals the product of the square roots of the factors

$$
\sqrt{a b}=\sqrt{a} \cdot \sqrt{b}
$$

Ex1) $\sqrt{400}=\sqrt{4} \cdot \sqrt{100}=2 \cdot 10=20 \quad$ Ex2) $\sqrt{75}=\sqrt{25} \cdot \sqrt{3}=5 \sqrt{3}$

Quotient Property: The square root of a quotient equals the quotient of the square roots of the numerator and denominator

$$
\begin{gathered}
\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}} \\
\text { Ex3) } \sqrt{\frac{9}{25}}=\frac{\sqrt{9}}{\sqrt{25}}=\frac{3}{5}
\end{gathered} \quad \text { Ex4) } \sqrt{\frac{27}{16}=\frac{\sqrt{27}}{\sqrt{16}}=\frac{\sqrt{9 \cdot 3}}{4}=\frac{\sqrt{9} \cdot \sqrt{3}}{4}=\frac{3 \sqrt{3}}{4}}
$$

Simplest Form: An expression with radicals is in simplest form if the following are true:

- no perfect square factors (other than 1) are in the radicand (under the radical): $\sqrt{8} \rightarrow \sqrt{4 \cdot 2} \rightarrow 2 \sqrt{2}$
- no fractions are in the radicand (under the radical): $\sqrt{\frac{5}{16}} \rightarrow \frac{\sqrt{5}}{\sqrt{16}} \rightarrow \frac{\sqrt{5}}{4}$
- no radicals appear in the denominator of the fraction: $\frac{1}{\sqrt{5}} \rightarrow \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} \rightarrow \frac{\sqrt{5}}{\sqrt{25}}=\frac{\sqrt{5}}{5}$
[This is called "rationalizing the denominator": http://www.khanacademy.org/video/how-to-rationalize-adenominator?playlist=ck12.org\ Algebra\ 1\ Examples ]

Simplify - leave answers in simplest radical from - no decimal answers.

1. $\sqrt{12}$
2. $10 \sqrt{250}$
3. $\sqrt{32}$
4. $\sqrt{128}$
5. $\sqrt{80}$
6. $\sqrt{48 x^{2} y^{6}}$
7. $\sqrt{24 x^{3}}$
8. $4 \sqrt{90}$
9. $\sqrt{\frac{1}{169}}$
10. $\sqrt{\frac{9}{81}}$
11. $\sqrt{\frac{9}{5}}$
12. $\frac{8}{\sqrt{2}}$

## 3 Linear Equations

A. Graphing. The graph of a linear equation is a line. Here is a quick review of how to graph common forms of a linear equation.

Form 1: Horizontal Line $y=c(\mathrm{c}=$ real number $)$
The graph of $y=c$ is a horizontal line that passes through the point $(0, c)$. The equation $y=5$ is simply graphed by moving on the $y$-axis to the point $(0,5)$ and then drawing a horizontal line (see figure 1 ). Likewise, the equation $y=0$ is graphed by going to the point $(0,0)$ and drawing a horizontal line - in this case, the graph is the $x$-axis (see figure 2 ).


Figure 1


Figure 2


Form 2: Vertical Line $x=c(\mathrm{c}=$ real number $)$
The graph of $x=c$ is a vertical line that passes through the point $(c, 0)$. The equation $x=-3$ is simply graphed by moving on the x -axis to the point $(-3,0)$ and then drawing a vertical line (see figure 3 ). Likewise, the equation $x=0$ is graphed by going to the point $(0,0)$ and drawing a vertical line - in this case, the graph is the $y$-axis.

Form 3: Slope-Intercept $y=m x+b \quad(m=$ slope, $b=y$-intercept $)$ The equation $y=2 x-3$ is in Slope-Intercept form. To graph this line, first plot the $y$-intercept $(0,-3)$. Then, recalling that the slope is the ratio of rise/run (2/1), find a second point on the line by going up 2 and then right 1 [to point $(1,-1)$ ]. Connect the points to graph the line (see figure 4).
http://www.brightstorm.com/math/algebra/linear-equations-and-their-graphs/how-to-graph-a-line-using-y-equals-mx-plus-b http://www.khanacademy.org/video/graphs-using-slope-interceptform?playlist=ck12.org\ Algebra\ 1\ Examples


Form 4: Standard Form $A x+B y=C$
The equation $2 x-3 y=12$ is in Standard form. To graph this equation transform this equation into its equivalent Slope-Intercept form $y=m x+b$. http://brightstorm.com/math/algebra/linear-equations-and-their-graphs/standard-form-of-linear-equations (see Problem 3)

Exercises: Graph each line in the space provided.


## B. Finding Equations of a Line.

## Given a slope and a point, find an equation of a line:

Ex7) Find an equation of the line with a slope of $2 / 3$ and containing the point $(6,-3)$.
Method I: Since the slope and a point are provided, use the Point-Slope form: $y-y_{1}=m\left(x-x_{1}\right)$
Step 1: Substitute for the given slope and point: $y-(-3)=2 / 3(x-6)$
Step 2: Simplify: $y+3=2 / 3(x-6)$

Method II: Use the Slope-Intercept form $y=m x+b$
Step 1: Substitute for the given slope: $y=(2 / 3) x+b$

$$
-3=(2 / 3)(6)+b
$$

Step 2: Find $b$ (y-intercept) by substituting in the point: $-3=4+b$

$$
-7=b
$$

Step 3: Substitute for $b$ into the equation from Step 1 and simplify: $y=2 / 3 x+(-7)=2 / 3 x-7$

## Given two points, find an equation of a line:

Ex8) Find an equation of a line containing the points $(-3,2)$ and $(7,4)$
Step 1: Find the slope: $m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}=\frac{4-2}{7-(-3)}=\frac{2}{10}=\frac{1}{5}$
Step 2: Use either Method I or Method II from Example 7 above. You may use either of the two given points.

## Method I (using 1 ${ }^{\text {st }}$ point).

$y-(2)=1 / 5(x-(-3))$
$y-2=1 / 5(x+3)$

## Method II (using 1 ${ }^{\text {st }}$ point).

$y=(1 / 5) x+b$
$2=1 / 5(-3)+b$
$2=-3 / 5+b$
$2+3 / 5=b$
$10 / 5+3 / 5=b$
$13 / 5=b \quad$ so, $y=1 / 5 x+13 / 5$
*NOTE: The above two linear equations may appear different; but, they are equivalent.
For the Method I equation: solve for $\mathrm{y}-$ - distribute the $1 / 5$ inside the parenthesis, then add 2 to the right side

$$
\begin{gathered}
y-2=\frac{1}{5}(x+3) \Rightarrow 5(y-2)=(x+3) \\
5 y-10=x+3 \Rightarrow 5 y=x+3+10 \\
5 y=x+13 \Rightarrow y=\frac{1}{5} x+13
\end{gathered}
$$

Find an equation of the line in slope intercept form with the given:

1. Slope of 2 and $y$-intercept of 1
2. Slope of $1 / 2$ and point $(12,1)$
3. Contains points $(4,-1)$ and $(1,2)$
4. Contains points $(5,1)$ and $(8,1)$
5. Slope of -4 and point $(-6,1)$
6. Contains points $(6,3)$ and $(8,9)$

Rate of Change. Find the slope $m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}=\frac{4-2}{7-(-3)}=\frac{2}{10}=\frac{1}{5}$
7.

Cardinal Mooney Sports Budget

| Year | Per-student budget |
| :---: | :---: |
| 2002 | $\$ 84$ |
| 2003 | $\$ 79$ |
| 2004 | $\$ 98$ |
| 2005 | $\$ 84$ |
| 2006 | $\$ 96$ |

According to the table, what was the rate of change between 2002 and 2006 ?
8. Find the average rate of change between the date from 3 to 8 .

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 81 | 83 | 80 | 95 | 88 | 91 | 60 | 93 | 85 |

9. Find the rate of change of the given line.


## 4. Polynomials - Add/Subtract, Multiply

Suggested videos on polynomials:
http://brightstorm.com/math/algebra/polynomials-2
http://brightstorm.com/math/algebra-2/polynomials/dividing-polynomials-using-long-division
http://www.khanacademy.org/video/addition-and-subtraction-of-polynomials?playlist=ck12.org\ Algebra\ 1\ Examples
http://www.khanacademy.org/video/multiplication-of-polynomials?playlist=ck12.org\ Algebra\ 1\ Examples
http://www.khanacademy.org/video/polynomial-division?playlist=ck12.org\ Algebra\ 1\ Examples
Addition: Add like terms

$$
\left(x^{3}+2 x^{2}-8 x\right)-\left(-2 x^{2}+7 x-5\right)=
$$

$$
\begin{array}{r}
(4 x+6 y)+(2 x-3 y)= \\
\text { Ex1) }(4 x+2 x)+(6 y-3 y)= \\
6 x+3 y
\end{array}
$$

Subtraction: Add the additive inverse

$$
\left(x^{3}+2 x^{2}-8 x\right)+\left(+2 x^{2}-7 x+5\right)=
$$

$$
\left(x^{3}\right)+\left(2 x^{2}+2 x^{2}\right)+(-8 x-7 x)+(5)=
$$

$$
x^{3}+4 x^{2}-15 x+5
$$

Multiplication: Use the distributive property.

$$
\begin{aligned}
7 y(-6 y-9) & =7 y(-6 y)+7 y(-9) \\
& =-42 y^{2}-63 y
\end{aligned}
$$

$$
(x+2)(x-5)=x^{2}-5 x+2 x-10
$$

$$
=x^{2}-3 x-10
$$

## *Recall FOIL (First-Outside-Inside-Last)

## SIMPLIFY: Circle answers

1. $\left(y^{2}+2 y-5\right)+\left(8 y^{2}-5 y+9\right)$
2. $\left(7 x^{3}-5 x^{2}-2\right)-\left(5 x^{3}-2 x^{2}+4\right)$
3. $-2 x y\left(6 x^{2}-4 x y+5 y^{2}\right)$
4. $(2 x-5)(3 x+2)$
5. $(2 x+3)(3 x+5)$
6. $(3 x-5)^{2}$

## 5. Factoring Polynomials

Suggested videos on factoring polynomials: http://brightstorm.com/math/algebra/factoring-2
http://www.khanacademy.org/video/factoring-quadratic-expressions?playlist=ck12.org\ Algebra\ 1\ Examples

Perfect Square Trinomials $a^{2}+2 a b+b^{2}=(a+b)^{2}$ and $a^{2}-2 a b+b^{2}=(a-b)^{2}$ http://brightstorm.com/math/algebra/factoring-2/factoring-special-cases-part-i

Ex1) $x^{2}+4 x+4=(x+2)(x+2)=(x+2)^{2} \quad$ Ex2) $y^{2}-14 y+49=(y-7)(y-7)=(y-7)^{2}$
Ex3) $4 x^{2}-4 x y+y^{2}=(2 x-y)(2 x-y)=(2 x-y)^{2}$

## Factor the following perfect square trinomials:

1. $x^{2}+6 x+9$
2. $y^{2}-16 y+64$
3. $x^{2}+4 x y+4 y^{2}$
4. $9 x^{2}-12 x y+4 y^{2}$

Trinomials $a x^{2}+b x+c, a=1$ (i.e., Lead Coefficient $=1$ ) Examples: $x^{2}+8 x+15$ and $x^{2}-4 x+3$ http://brightstorm.com/math/algebra/factoring-2/factoring-trinomials-a-equals-1

Example: Factor $x^{2}+2 x-15$
Step 1: Find the square root of the first term. The square root of the first term is $x$.
Step 2: Find all factors of the third term, -15 . The factors of the third term are: $\{-3,5\},\{3,-5\},\{-1,15\},\{1,-15\}$
Step 3: Decide which of these factors can be added to find the coefficient of the middle term.

$$
5+(-3)=2 \text {, so }\{-3,5\} \text { are the two factors needed to get the middle term. }
$$

Therefore, $(x-3)(x+5)$ are the two binomial factors. Check the answer by Foiling: $(x-3)(x+5)=x^{2}+2 x-15$
Ex4) $x^{2}+8 x+15=(x+5)(x+3)$
Ex5) $x^{2}-4 x+3=(x-3)(x-1)$

Ex6) $y^{2}+10 y-11=(y+11)(y-1)$

## Factor the following trinomials:

5. $x^{2}+8 x+7$
6. $y^{2}-7 y+12$
7. $x^{2}-7 x+10$
8. $y^{2}+9 y-20$

Difference of Two Squares $a^{2}-b^{2}=(a+b)(a-b)$ Examples: $x^{2}-16$ and $4 x^{2}-25 y^{2}$
http://brightstorm.com/math/algebra/factoring-2/factoring-special-cases-part-i
Example: Factor $x^{2}-y^{2}$
Step 1: Find the square root of each term: $\sqrt{x^{2}}=x, \sqrt{y^{2}}=y$
Step 2: The first factor will be the SUM of these two square roots: $(x+y)$
Step 3: The second factor will be the DIFFERENCE of these two square roots: $(x-y)$
Therefore, $x^{2}-y^{2}=(x+y)(x-y)$

Ex11) $x^{2}-16=(x+4)(x-4)$
Ex12) $4 x^{2}-25 y^{2}=(2 x-5 y)(2 x+5 y)$

Ex13) $8 x^{2}-32=8\left(x^{2}-4\right)=8(x+2)(x-2)$
Ex14) $4-9 x^{2}=(2-3 x)(2+3 x)$

Factor the following difference of two squares:
13. $x^{2}-49$
14. $81 y^{2}-1$
15. $100-9 y^{2}$
16. $9 x^{2}-64 y^{2}$

## 6. Rules of Exponents

At the start of each section, you may be provided some links to websites that have short videos that explain concepts and provide examples. Here are the suggested links to help on the first section, Rules of Exponents:
http://brightstorm.com/math/algebra/exponents
http://www.khanacademy.org/video/exponent-rules-1?playlist=Algebra\ 1\ Worked\ Examples
http://www.khanacademy.org/video/exponent-rules-2?playlist=Algebra\ 1\ Worked\ Examples
http://www.khanacademy.org/video/exponent-rules-3?playlist=Algebra\ l\ Worked\ Examples

## Product of Powers Property

When multiplying factors together that have the same base, you must multiply coefficients together, then multiply variables. When you multiply the same variables, you keep the variable and add the exponents together.
Ex1) $2^{2} \cdot 2^{3}=2^{2+3}=2^{5}$
Ex2) $x^{4} \cdot x=x^{4+1}=x^{5}$
Ex3) $x^{2} \cdot x^{4} \cdot x=x^{2+4+1}=x^{7}$
Ex4) $-2 x^{7} \cdot 4 x^{3}=(-2 \cdot 4)\left(x^{7}+x^{3}\right)=-8 x^{10}$

Directions: Use the product of powers property to simplify the expression:

1. $x \bullet x^{3} \bullet x^{4}$
2. $4 \bullet 4^{3} \bullet 4^{8}$
3. $y^{3} \bullet y \bullet y^{5}$
4. $\left(x^{2} y\right)\left(x^{4} y^{5}\right)(x y)$

## Power of a Power Property

To find the power of a power, multiply the exponents (remember, power outside parentheses - applies to each base inside the parentheses)
Ex4) $\left(z^{4}\right)^{5}=z^{4 \cdot 5}=z^{20} \quad$ Ex5) $\left(2 x^{2}\right)^{4}=2^{4} \bullet\left(x^{2}\right)^{4}=16 x^{8}$

Ex6) $\left(-2 x^{3}\right)^{4}=(-2)^{4} \bullet\left(x^{3}\right)^{4}=16 x^{12}$

Directions: Use the power of a power property to simplify the expression:
5. $(-5)^{3}$
6. $\left(x y^{2}\right)^{3}$
7. $\left(x^{5}\right)^{6}$
8. $(x y)^{2}$
9. $\left(x^{6} y^{3}\right)^{3}$
10. $\left(-2 z^{4}\right)^{2}$

## Power of a Product Property

To find a power of a product, find the power of each factor and multiply.
Ex7) $(-4 x y)^{2}=(-4 \bullet x \bullet y)^{2}=(-4)^{2} \bullet x^{2} \bullet y^{2}=16 x^{2} y^{2}$

Directions: Use the power rules to simplify the following expressions:
11. $\left[(6)^{2}\right]^{3}$
12. $\left[\left(3 x y z^{2}\right)^{3}\right]^{4}$
13. $(x)^{4} \bullet(-4 x)^{3}$
http://www.brightstorm.com/math/algebra/exponents/zero-and-negative-exponents
http://www.khanacademy.org/video/zero--negative--and-fractional-exponents?playlist=ck12.org\ Algebra\ 1\ Examples

## Zero Exponents

A base that has a " 0 " as an exponent can be evaluated to have a value of 1: $a^{0}=1$

## Negative Exponents

$$
a^{-n} \text { is the reciprocal of } a^{n}: a^{-n}=\frac{1}{a^{n}}
$$

If a negative exponent is in the denominator, then, it can be rewritten as: $\frac{1}{a^{-n}}=a^{n}$
Ex8) Rewrite with no zero exponents or negative exponents.
a) $(-8)^{0}=1$
b) $4^{-2}=\frac{1}{4^{2}}=\frac{1}{16}$
c) $5 y^{-1} z^{-2}=5 \bullet \frac{1}{y} \bullet \frac{1}{z^{2}}=\frac{5}{y z^{2}}$
d) $\frac{1}{(2 x)^{-4}}=(2 x)^{4}=2^{4} x^{4}=16 x^{4}$

Directions: Simplify the following expressions using the law of exponents. Make sure to rewrite with no zero or negative exponents.
14. $(y)^{-2}$
15. $(2 c)^{-4}$
16. $8^{-1} \bullet 8$
17. $4^{6} \bullet 4^{-4}$
18. $\left(5^{-2}\right)^{2}$
19. $x^{3} y^{-6}$
http://www.khanacademy.org/video/exponent-properties-involving-quotients?playlist=ck12.org\ Algebra\ 1\ Examples
Quotient of Powers Property
When you are dividing "like bases", you may cancel common factors (separated by multiplication) from both the numerator and the denominator.
Ex9) $\frac{x^{4}}{x^{2}}=\frac{x \bullet x \bullet x \bullet x}{x \bullet x}=\frac{x^{2}}{1}=x^{2} \quad \quad \mathbf{E x 1 0 )} \frac{y^{2}}{y^{3}}=\frac{y \bullet y}{y \bullet y \bullet y}=\frac{1}{y}$
Notice - in the original example and the answer, you obtained the exponent in the answer by "subtracting the exponents" for the like bases. This is your rule for dividing like bases-you subtract the exponents.

Directions: Simplify the following by using the law of exponents:
20. $\frac{10^{5}}{10^{3}}$
21. $\frac{3}{x^{2}} \cdot 2 x^{4}$
22. $\frac{3 x^{4} y^{2}}{9 x^{6} y^{4}}$
23. $\frac{x^{4} \cdot x^{5}}{x^{16}}$
24. $\left(\frac{2}{3}\right)^{3}$
25. $\frac{(2 x y)^{3}}{x^{2}}$

