SUMMER ASSIGNMENT

AP CALCULUS

CARDINAL MOONEY HIGH SCHOOL

2023-24 ACADEMIC YEAR

INSTRUCTOR | MR. RYAN KRAUSE

CONTACT | RKRAUSE@CMHS-SARASOTA.ORG

Dear Incoming AP Calculus Students:

Congratulations on choosing to take AP Calculus AB! This is an extremely rigorous but rewarding course that will truly prepare you for further courses in college mathematics. Don't be scared – you would not have been recommended for this course if I didn't feel you were ready for it!

This summer assignment is meant to ensure you have down the essential skills needed to enter this course. Nothing in this packet is new material – you already have experience with it all. Overall, this review assignment should take you approximately 6-8 hours to complete throughout the summer. I would suggest that you spread the work of the assignment out over the course of the summer. Do not wait until one week before the end of the summer to get started!

You would not be taking this course if you didn't have a passion for mathematics and want to be here. That being said, I expect that all work you submit in this course is done honestly and to the best of your ability. If you have any instinct to use PhotoMath or another source to quickly get yourself through this assignment, or if you feel you should wait until a couple days before school starts to begin this assignment, then AP Calculus is not the course for you. You are only going to get out of this course (i.e. your grade on the AP Exam) what you put into it (i.e. your work throughout the year).

Students in AP Calculus are required to own a graphing calculator. My recommendation is that you own a TI-84 Plus CE as that is the model that I will be using for all my in-class demonstrations and is the one that is most commonly used. However, you are technically permitted to use any calculator on the AP Approved Calculator list.

Your grade for this course will be determined as follows: 50% test grades, 30% quiz grades, 10% skill builders, and 10% homework. Assignments in the "homework" category will be graded for completion, while assignments in the "skill builder" category will be graded for accuracy.

This summer assignment will be counted in the skill builder category. The assignment will be worth 50 points, some of which will be based off completion, and some of which will be based off your accuracy of some randomly selected problems.

Completeness Score Is the assignment fully completed? Are <u>all</u> answers supported by adequate work?	30 points
Accuracy Score Twenty random problems from this assignment will be checked for accuracy. Each will be worth one point.	20 points

If you find that there are topics you are struggling with on this assignment, I encourage you to either look back at notes from prior courses or consult the vast number of online resources at your disposal. Also, I will host Zoom sessions on the following dates where anyone is welcome to join and ask any questions they may have as they work through this assignment. You can join each Zoom session by scanning the QR codes below.

Zoom Session #1	**Zoom Session #2**	**Zoom Session #3**
June 21st 7-8 PM	July 12th 7-8 PM	August 2nd 7-8 PM

You are also welcome to email me over the summer with questions. Please include a screenshot of your work so that I can see where you're at and best help you.

This assignment will be due on the first day of classes, August 10th. You may either submit the assignment to Canvas (if working in Notability), or you may submit a hard copy of the assignment. If submitting to Canvas, you may do so in-class on Thursday, August 10th. If working out the assignment on paper, please bring your completed version to class on Thursday, August 10th.

Have a great summer!

Mr. Krause

FUNCTIONS

1. If $f(x) = 4x - x^2$, find:

(a) $f(4) - f(-4)$

(b) $\sqrt{f\left(\frac{3}{2}\right)}$

2. If $V(r) = \frac{4}{3}\pi r^3$, find:

(a) $V\left(\frac{3}{4}\right)$

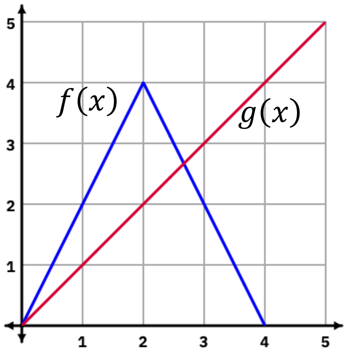
(b) $V(r+1) - V(r-1)$

(c) $\frac{V(2r)}{V(r)}$

3. Given the graphs of f and g in the image to the right, find:

(a) $(f - g)(3)$

(b) $f(g(3))$



4. If $f(x) = \begin{cases} -x & x < 0 \\ x^2 - 1 & 0 \leq x < 2 \\ \sqrt{x+2} - 2 & x \geq 2 \end{cases}$, find:

(a) $f(0) - f(2)$

(b) $\sqrt{5 - f(-4)}$

(c) $f(f(3))$

DOMAIN & RANGE

Write the domain of the following functions using interval notation.

1. $f(x) = 3$

2. $y = x^3 - x^2 + x$

3. $y = \frac{x^3 - x^2 + x}{x}$

4. $y = \frac{x-4}{x^2-16}$

5. $f(x) = \frac{1}{4x^2-4x-3}$

6. $y = \sqrt{2x - 9}$

7. $y = \log(x - 10)$

8. $y = \frac{\sqrt{2x+14}}{x^2-49}$

Write the range of the following functions in interval notation.

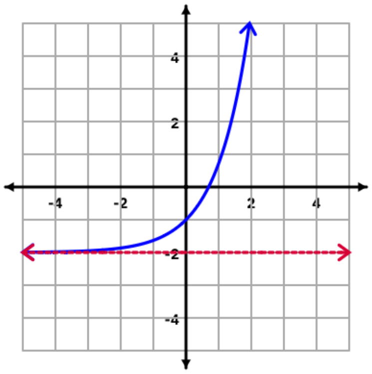
9. $y = x^4 + x^2 + 1$

10. $y = 100^x$

11. $y = \sqrt{x^2 + 1} + 1$

Write the domain and range of the following functions in interval notation.

12.



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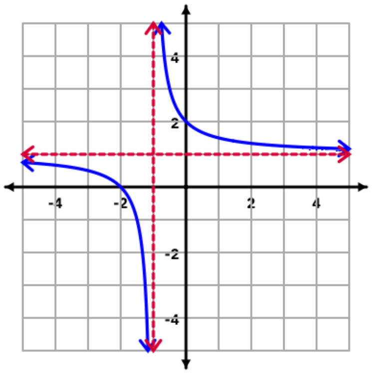

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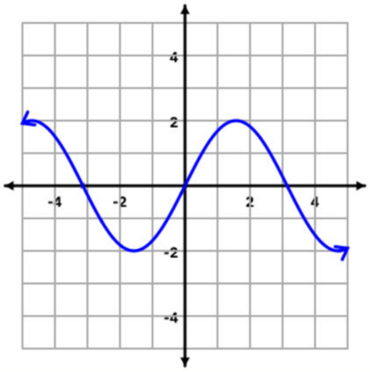
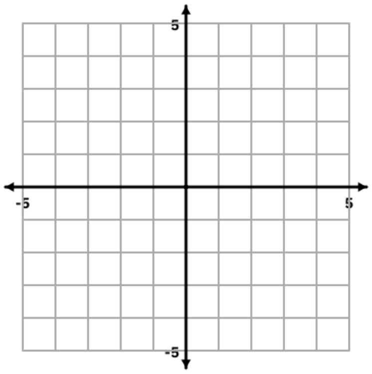


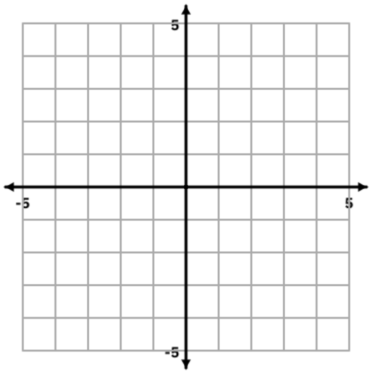
GRAPHS OF FUNCTIONS

Graph each of the following functions as accurately as possible. You will need to be <u>very</u> familiar with these graphs throughout AP Calculus. If needed, you may use a graphing calculator to assist you on some of them.

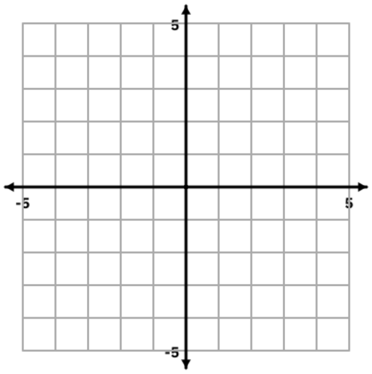
1. $y = x$



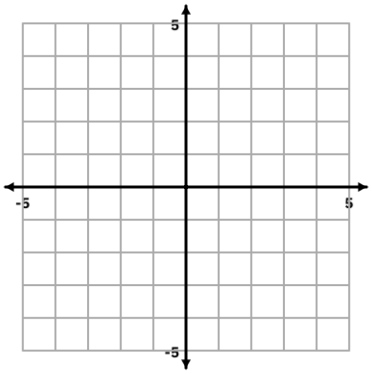
2. $y = x^2$



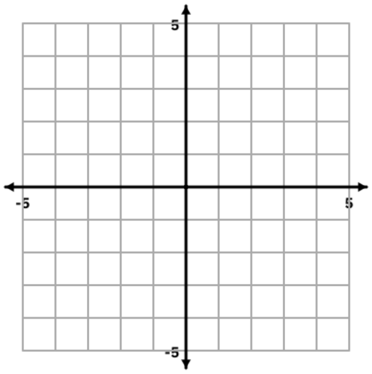
3. $y = x^3$



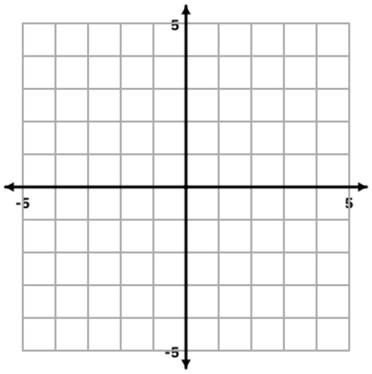
4. $y = \sqrt{x}$



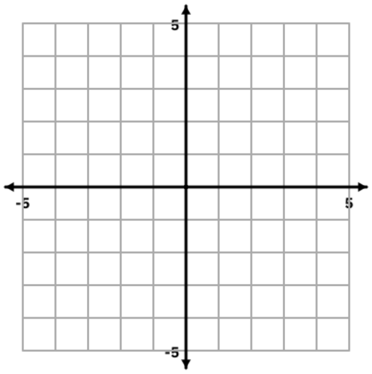
5. $y = |x|$



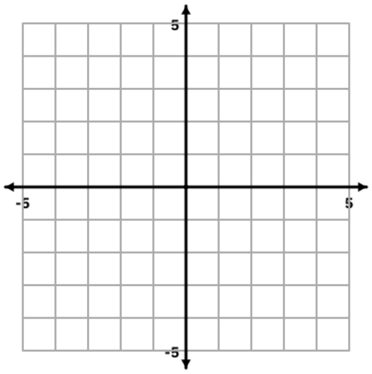
6. $y = \frac{|x|}{x}$



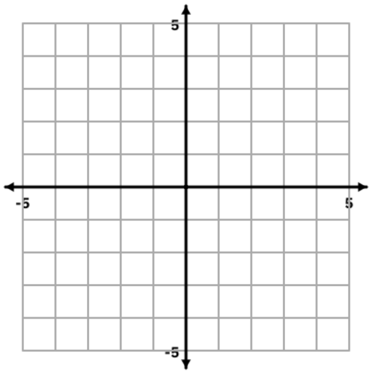
7. $y = e^x$



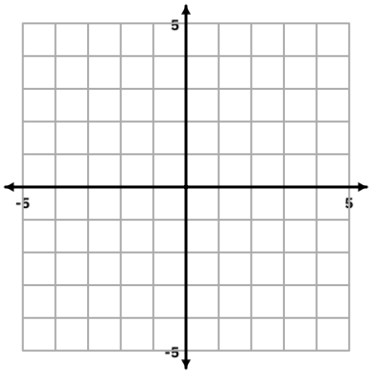
8. $y = \ln x$



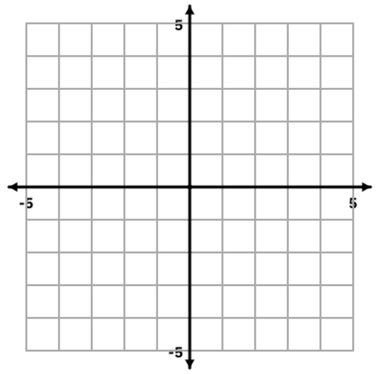
9. $y = \frac{1}{x}$



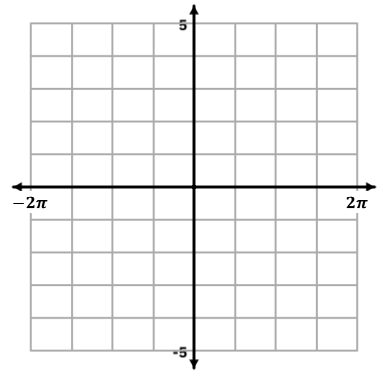
10. $y = \frac{1}{x^2}$



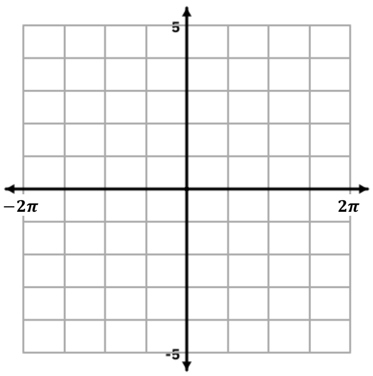
11. $y = \sqrt{4 - x^2}$



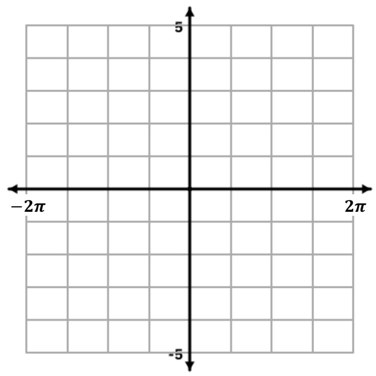
12. $y = \sin x$



13. $y = \cos x$



14. $y = \tan x$



TRANSFORMATIONS

1. For any function $f(x)$, describe in words what the following would do to the graph of $f(x)$.

(a) $f(x) - 4$

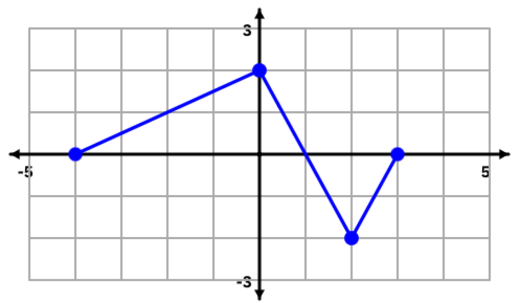
(b) $f(x - 4)$

(c) $-f(x + 2)$

(d) $5f(x) + 3$

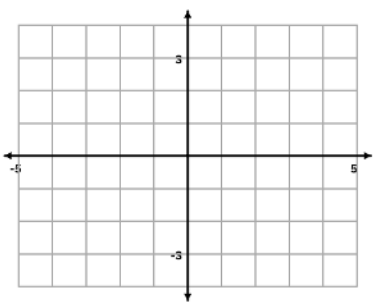
(e) $f(2x)$

(f) $|f(x)|$

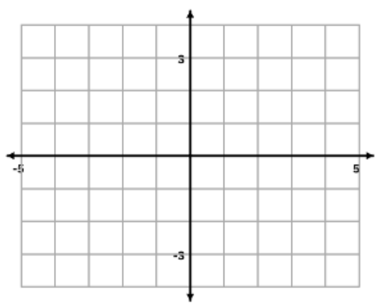


2. Use the graph of $f(x)$ above to sketch the graphs below.

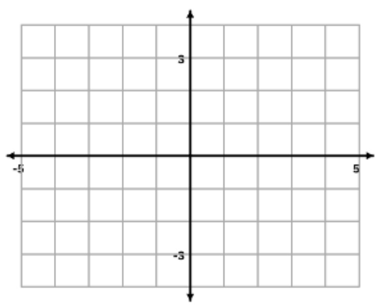
(a) $y = 2f(x)$



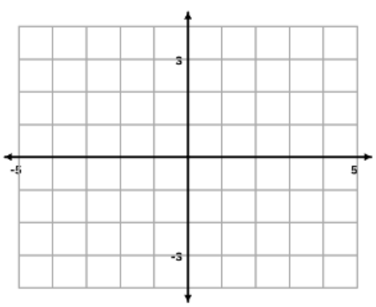
(b) $y = -f(x)$



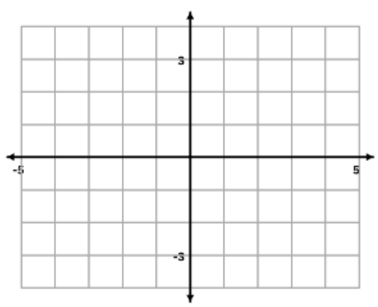
(c) $y = f(x - 1)$



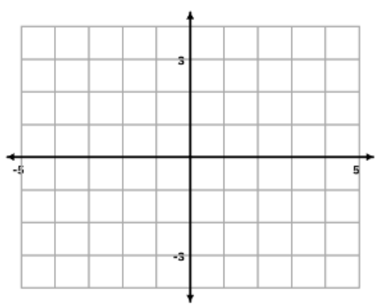
(d) $y = f(x) + 2$



(e) $y = |f(x)|$



(f) $y = f(|x|)$



FACTORING

Factor each expression below completely.

1. $x^2 - 16x + 63$

2. $7m^2 - 31m - 20$

3. $28n^4 + 16n^3 - 80n^2$

4. $x^3 + 8$

5. $x^3 - 8$

6. $x^4 + 11x^2 - 80$

7. $9x^4 - 81$

8. $4r^2 - \frac{1}{4}$

9. $30n^2b - 87nb + 30b$

LINEAR FUNCTIONS

1. Find the equation of the line in point-slope form with the given slope, passing through the given point.

(a) $m = -7, \ (-3, -7)$

(b) $m = \frac{2}{3}, \ \left(-6, \frac{1}{3}\right)$

2. Find the equation of the line in point-slope form passing through the given points

(a) $(-3, 6), (-1, 2)$

(b) $\left(-2, \frac{2}{3}\right), \left(\frac{1}{2}, 1\right)$

3. Find the equation of the line in point-slope form that is **(a)** parallel, and **(b)** normal, to the line $5x + 2y = 7$ and passes through the point $(-6, 2)$.

4. Find k if the lines $3x - 5y = 9$ and $2x + ky = 11$ are parallel.

SOLVING POLYNOMIAL EQ.

For each equation, find all real solutions x.

1. $x^2 + 7x - 18 = 0$

2. $x^2 + x + \frac{1}{4} = 0$

3. $2x^2 - 72 = 0$

4. $12x^2 - 5x = 2$

5. $20x^2 - 56x + 15 = 0$

6. $81x^2 + 72x + 16 = 0$

7. $x + \frac{1}{x} = \frac{17}{4}$

8. $x^3 - 5x^2 + 5x - 25 = 0$

9. $2x^4 - 15x^3 + 18x^2 = 0$

RATIONAL FUNCTIONS

For each function, identify any and all vertical asymptotes, horizontal asymptotes, and holes (if they exist). Be sure to write all asymptotes as <u>equations</u>, and all holes as <u>coordinate points</u>.

1. $y = \frac{x-1}{x+5}$

2. $y = \frac{2x+16}{x+8}$

3. $y = \frac{2x^2+16}{x^2+5x+6}$

4. $y = \frac{x}{x^2-25}$

5. $y = \frac{x^3}{x^2+4}$

6. $y = \frac{10x+20}{x^3-2x^2-4x+8}$

EXPONENTS

Simplify. Your answer should contain only positive exponents.

1. $(2v)^2 \cdot 2v^2$

2. $\frac{x^3y^3 \cdot x^3}{4x^2}$

3. $\frac{2y^3 \cdot 3xy^3}{3x^2y^4}$

4. -12^2x^{-5}

5. $(-12x^5)^{-2}$

6. $\left(\frac{-4}{x^4}\right)^{-3}$

7. $(x^3 - 1)^{-2}$

8. $(121x^8)^{1/2}$

9. $(-32x^{-5})^{-3/5}$

FRACTIONS

Simplify as much as possible. Express all final answers with improper fractions.

1. $\frac{5}{4} + \frac{6}{7}$

2. $4\left(\frac{2}{5}\right) \div \frac{8}{9}$

3. $\left(\frac{9}{2}\right)^2$

4. $\dfrac{\frac{5}{8}}{-\frac{2}{3}}$

5. $\dfrac{4 - \frac{2}{9}}{3 + \frac{4}{3}}$

6. $\dfrac{2 + \frac{7}{2} + \frac{3}{5}}{5 - \frac{3}{4}}$

7. $5(3)^2 - \left(\frac{2}{5}\right)^3$

8. $3\left(\frac{4}{7}\right)^{-2}$

9. $\dfrac{x - \frac{1}{x}}{x + \frac{1}{x}}$

INVERSE FUNCTIONS

Find the inverse of each function. Use a graphing utility to verify graphically that they are inverses.

1. $2x - 6y = 1$

2. $y = ax + b$

3. $y = 9 - x^2, \ x \geq 0$

4. $y = \sqrt{1 - x^3}$

5. $y = \frac{9}{x}$

6. $y = \frac{2x + 1}{3 - 2x}$

7. Find the inverse of the function below and prove that f and f^{-1} are inverses using function composition.

$$f(x) = \frac{x^2}{x^2 + 1}$$

Simplify the following:

1. $\log_2\left(\frac{1}{4}\right)$

2. $\log_8 4$

3. $\ln\left(\frac{1}{\sqrt[3]{e^2}}\right)$

4. $5^{\log_5 40}$

5. $e^{\ln 12}$

6. $\log_{12} 2 + \log_{12} 9 + \log_{12} 8$

7. $\log_2\left(\frac{2}{3}\right) + \log_2\left(\frac{3}{32}\right)$

8. $\log_{1/3}\left(\frac{4}{3}\right) - \log_{1/3} 12$

9. $\log_3\left(\sqrt{3}\right)^5$

SOLVING VARIOUS EQ.

Solve each equation for x. Be sure to check for extraneous solutions, if necessary.

1. $4|x-8| = 20$

2. $|8+2x| + 2x = 40$

3. $\sqrt{110-x} = x$

4. $-x + \sqrt{6x+19} = 2$

5. $\frac{2}{3} - \frac{1}{x} = \frac{5}{6}$

6. $\frac{1}{x-3} + \frac{1}{x+3} = \frac{10}{x^2-9}$

7. $\log_5(3x-8) = 2$

8. $\log_2(x-1) + \log_2(x+3) = 5$

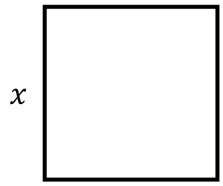
9. $3^{x-2} = 18$

10. $8^x = 5^{2x-1}$

GEOMETRY

Identify each of the following formulas, which will be used extensively in AP Calculus. Write all equations in terms of the variables provided in each diagram.

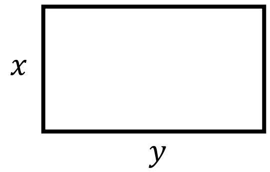
Square



Perimeter = ________________

Area = ________________

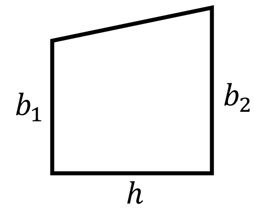
Rectangle



Perimeter = ________________

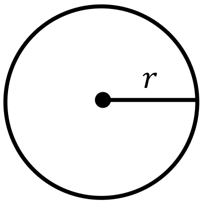
Area = ________________

Trapezoid



Area = ________________

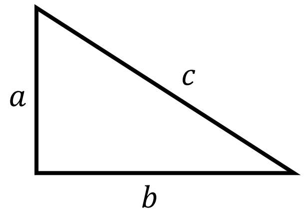
Circle



Circumference = ________________

Area = ________________

Triangle

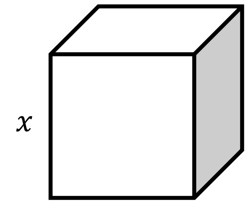


Pythagorean Thm (for right triangles)

____

Area (of any triangle) = ____________

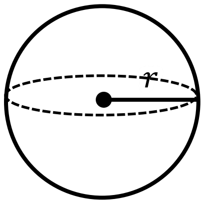
Cube



Volume = ________________

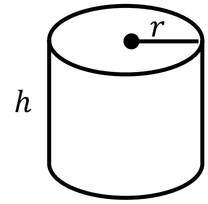
Surface Area = ________________

Sphere



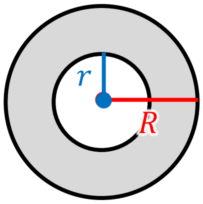
Volume = ________________

Cylinder



Volume = ________________

Washer

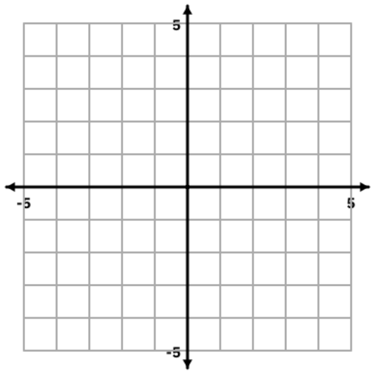


Area of Shaded Region =

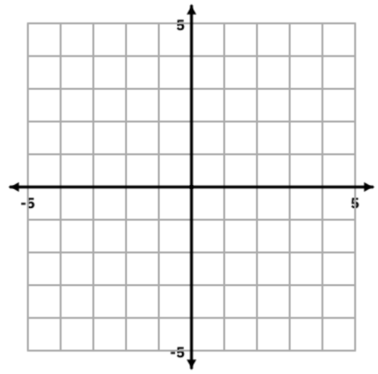
____

Find the area between the x-axis and $f(x)$ from x=0 to x=5. Sketch the region to assist you in visualizing the region.

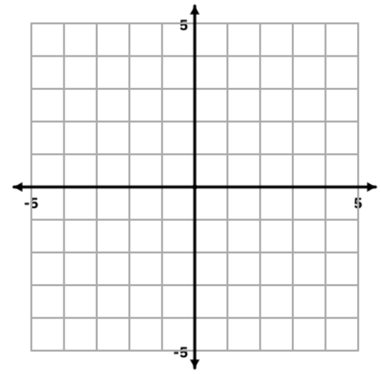
1. $f(x) = x + 3$



2. $f(x) = \sqrt{9 - x^2}$



3. $f(x) = \begin{cases} x + 1, & x \leq 2 \\ 5 - x, & x > 2 \end{cases}$



TRIGONOMETRY

TOPIC 0

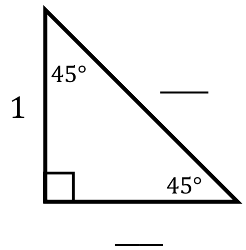
1. Evaluate without using a calculator.

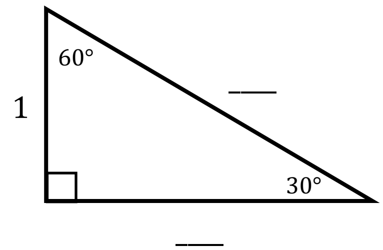
(a) $\cos\frac{2\pi}{3} =$ **(b)** $\tan\pi =$ **(c)** $\sin\left(-\frac{\pi}{4}\right) =$

(d) $\sec\frac{\pi}{2} =$ **(e)** $\tan\frac{7\pi}{4} =$ **(f)** $\cos\frac{7\pi}{6} =$

(g) $\cot\left(-\frac{\pi}{2}\right) =$ **(h)** $\sin 5\pi =$ **(i)** $\csc 0 =$

2. Fill in the blanks:





3. The point $P(-2, 4)$ is on the terminal side of an angle θ in standard position. Find all six trigonometric functions of the angle θ. (Your answers do not need to be rationalized.)

4. If $\cos\theta = -\frac{5}{13}$ and θ is in Quadrant II, find $\sin\theta$ and $\tan\theta$.

5. Identify the quadrant in which each of the following are true.

(a) $\sin\theta > 0$ and $\cos\theta < 0$ **(b)** $\csc\theta < 0$ and $\cot\theta > 0$ **(c)** $\tan\theta > 0$ and $\sec\theta < 0$

TRIGONOMETRIC EQUATIONS & IDENTITIES

Solve each equation on the interval $[0, 2\pi)$. Do not use a calculator.

1. $\sin^2\theta = \sin\theta$

2. $3\tan^3\theta = \tan\theta$

3. $3\sqrt{2}\cos\theta + 2 = -1$

4. $2\sin^2\theta - 3\sin\theta + 1 = 0$

5. $\sin 2\theta = \frac{1}{2}$

Fill in the blanks to complete each identity.

6. $\sin^2\theta + \cos^2\theta =$ _____

7. $1 +$ ____________ $= \sec^2\theta$

8. $1 + \cot^2\theta =$ ____________

9. $\sec\theta = \dfrac{1}{\_\_\_\_\_\_\_\_\_\_}$

10. $\dfrac{\sin\theta}{\cos\theta} =$ ____________

11. $\dfrac{1}{\csc\theta} =$ ____________

GRAPHING UTILITY

One-third of the AP Calculus AB exam is "graphing calculator required." You must be very well-versed in how to use the features of your graphing calculator. In this course, I will use a TI-84 Plus C, but you are technically welcome to use any calculator on College Board's Approved Calculator List.

When using a calculator to obtain an answer on an AP question, you must round your solution to <u>three decimal places</u> correctly or you will not earn credit for that answer.

1. Use a graphing utility to find the zero(s) of each function. Be sure to set each equation equal to zero first.

(a) $3x^3 - x - 5 = 0$

(b) $2x^2 - 1 = 2^x$

(c) $2\ln(x+1) = 5\cos x$ on $[0, 2\pi)$

2. Find the solution (intersection) of the following system of equations.

$$\begin{cases} f(x) = x^4 - 6.5x^2 + 6x + 2 \\ g(x) = 1 + x + e^{x^2 - 2x} \end{cases}$$

3. Use your graphing utility to find all relative maximum and minimum points of the given function.

$h(x) = 2x^5 - 3x^4 + x - 4$